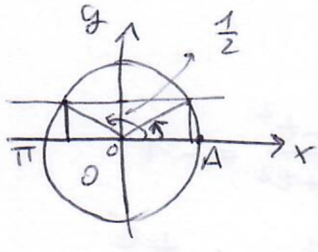
SOLUZIONE VERIFICA 18-10-2017 ①

1) $8\sin x - 4 = 0 \qquad \sin x = \frac{\cancel{4}}{\cancel{8}_2} = \frac{1}{2}$



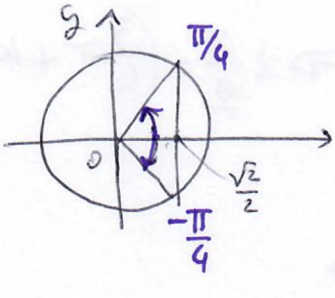
$$\boxed{x = \frac{\pi}{6} + 2k\pi} \quad \vee$$

$$x = \pi - \frac{\pi}{6} + 2k\pi = \boxed{\frac{5}{6}\pi + 2k\pi}$$

$$S = \left\{ \frac{\pi}{6} + 2k\pi, \frac{5}{6}\pi + 2k\pi \right\}$$

2) $3\cos x - \sqrt{2} = \cos x \rightarrow 3\cos x - \cos x = \sqrt{2} \qquad 2\cos x = \sqrt{2}$

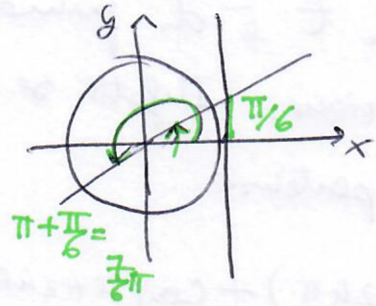
$$\cos x = \frac{\sqrt{2}}{2}$$



$$\boxed{\begin{array}{c} x = \frac{\pi}{4} + 2k\pi \\ \vee \\ x = -\frac{\pi}{4} + 2k\pi \end{array}}$$

3) $5\tan x - \sqrt{3} = 2\tan x \rightarrow 5\tan x - 2\tan x = \sqrt{3} \qquad 3\tan x = \sqrt{3}$

$$\tan x = \frac{\sqrt{3}}{3}$$



$$\boxed{x = \frac{\pi}{6} + k\pi}$$

4) $\sin 2x = \frac{1}{2} \qquad t = 2x \qquad \sin t = \frac{1}{2}$

$t = \frac{\pi}{6} + 2k\pi \rightarrow 2x = \frac{\pi}{6} + 2k\pi$

$x = \frac{\pi}{12} + k\pi$

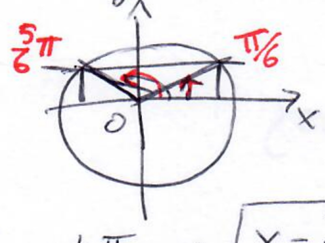
$\vee$

$t = \frac{5}{6}\pi + 2k\pi$

$\frac{\cancel{2}x}{\cancel{2}} = \left(\frac{5}{6}\pi + 2k\pi\right)\frac{1}{2}$

$x = \frac{5}{12}\pi + k\pi$

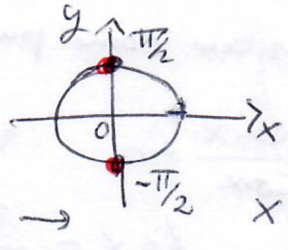
oppure



$$\frac{\cancel{2}x}{\cancel{2}} = \frac{\frac{\pi}{6} + 2k\pi}{2} \rightarrow \boxed{x = \frac{\pi}{12} + k\pi}$$

$$2x = \pi - \frac{\pi}{6} + 2k\pi \rightarrow \frac{\cancel{2}x}{\cancel{2}} = \frac{\frac{5}{6}\pi + 2k\pi}{2} \rightarrow \boxed{x = \frac{5}{12}\pi + k\pi}$$

5) $\cos\left(x + \frac{\pi}{6}\right) = 0$



$S: \boxed{x = \pm\frac{\pi}{3} + 2k\pi}$

$x + \frac{\pi}{6} = \frac{\pi}{2} + 2k\pi \longrightarrow x = \frac{\pi}{2} - \frac{\pi}{6} + 2k\pi = \frac{\cancel{2}^{1}}{\cancel{6}_{3}}\pi + 2k\pi$

$\vee$

$x + \frac{\pi}{6} = -\frac{\pi}{2} + 2k\pi \longrightarrow x = -\frac{\pi}{2} - \frac{\pi}{6} + 2k\pi = \frac{-\cancel{4}^{2}}{\cancel{6}_{3}}\pi + 2k\pi$

EQUAZIONE LINEARE COMPLETA IN SENO E COSENO

6) $\sin x + \cos x = -1$

FORMULE PARAMETRICHE

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2}$$

con $t = tg\frac{x}{2}$

$$\frac{x}{2} \neq \frac{\pi}{2} + k\pi$$

$$\boxed{x \neq \pi + 2k\pi}$$

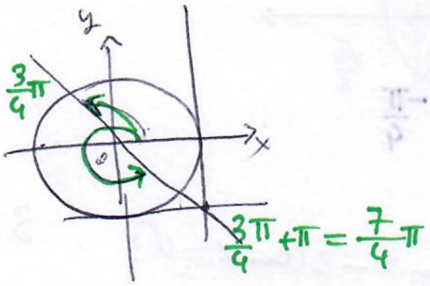
$$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = -1$$

$$2t + 1 \cancel{-t^2} = -1 \cancel{-t^2}$$

$$2t = -2$$

$$t = -1$$

$$tg\frac{x}{2} = -1 \Rightarrow \cancel{2}\frac{x}{\cancel{2}} = \left(\frac{3}{4}\pi + k\pi\right)2 \qquad \boxed{x = \frac{3}{2}\pi + 2k\pi}$$



Poiché l'equazione in t è di primo grado allora $x = \pi + 2k\pi$ è soluzione. Infatti se vado a sostituire nell'equazione di partenza:

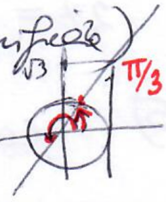
$$sen(\pi + 2k\pi) + \cos(\pi + 2k\pi) = -1$$

$$0 + (-1) = -1 \quad \underline{\text{vera}}$$

$$S = \left\{\frac{3}{2}\pi + 2k\pi, \ \pi + 2k\pi\right\}$$

7) $\sqrt{3}\sin x - 3\cos x = 0$ EQUAZIONE LINEARE OMOGENEA (SENZA TERMINE NOTO)

<u>Divido per $\cos x$</u>, che risulta $\neq 0$ per $x \neq \frac{\pi}{2} + k\pi$.
($x = \frac{\pi}{2} + k\pi$ non è soluzione come puoi facilmente verificare)



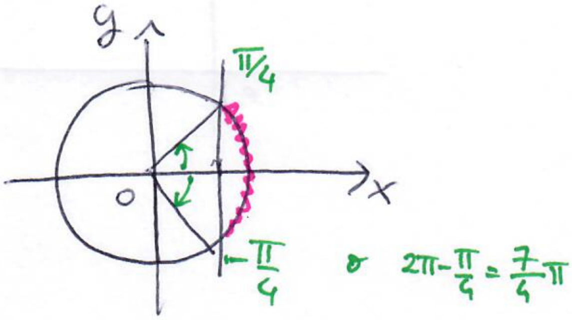
$$\sqrt{3}\frac{\sin x}{\cos x} - \frac{3\cos x}{\cos x} = 0 \qquad \sqrt{3}\, tg\, x - 3 = 0$$

$$tg\, x = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \qquad tg\, x = \frac{\cancel{3}\sqrt{3}}{\cancel{3}} \qquad tg\, x = \sqrt{3} \qquad \boxed{x = \frac{\pi}{3} + k\pi}$$

2) Risolvi le disequazioni

a) $2\cos x - \sqrt{2} \geq 0$

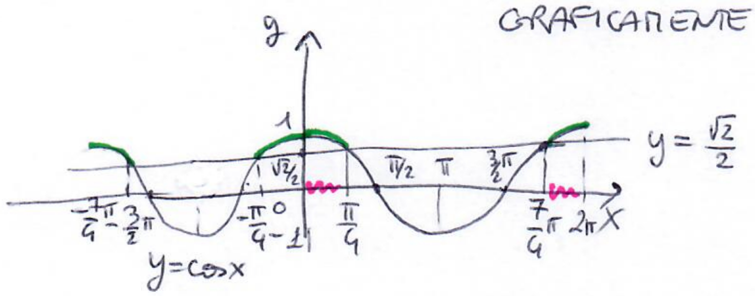
$\cos x \geq \frac{\sqrt{2}}{2}$



$-\frac{\pi}{4} + 2k\pi \leq x \leq \frac{\pi}{4} + 2k\pi$

(oppure $0 + 2k\pi \leq x \leq \frac{\pi}{4} + 2k\pi \;\vee\; -\frac{\pi}{4} + 2k\pi \leq x \leq 0 + 2k\pi$)

GRAFICAMENTE



Rappresentiamo nello stesso piano cartesiano $y = \cos x$ e $y = \frac{\sqrt{2}}{2}$ e vediamo che si intersecano in $x = \frac{\pi}{4} + 2k\pi$ e $x = \frac{7}{4}\pi + 2k\pi$

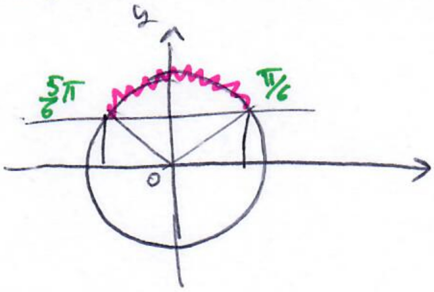
La funzione coseno è $\geq$ (ogni) di $\frac{\sqrt{2}}{2}$ negli intervalli

$0 \leq x \leq \frac{\pi}{4}$ $\quad \frac{7}{4}\pi \leq x \leq 2\pi$

se considero l'intervallo $[0, 2\pi]$

Altrimenti: $\boxed{-\frac{\pi}{4} + 2k\pi \leq x \leq \frac{\pi}{4} + 2k\pi}$

b) $1 - 2\sin x \leq 0$ $\quad -2\sin x \leq -1$ $\quad 2\sin x \geq 1$ $\quad \sin x \geq \frac{1}{2}$

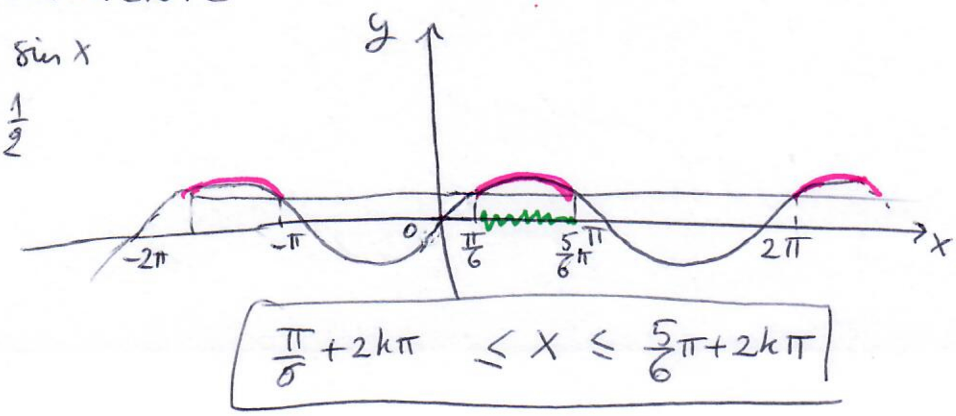


$\frac{\pi}{6} + 2k\pi \leq x \leq \frac{5}{6}\pi + 2k\pi$

GRAFICAMENTE

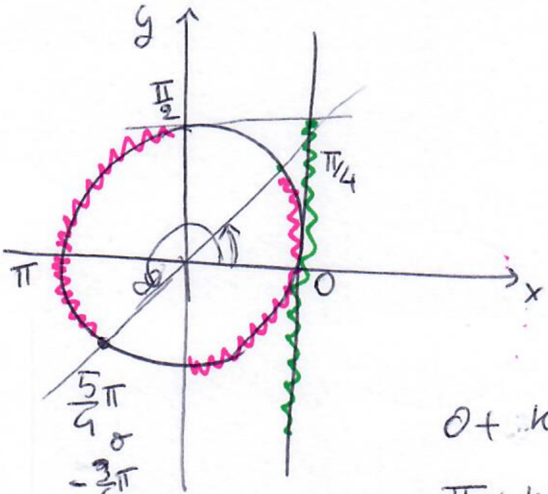
$y = \sin x$

$y = \frac{1}{2}$



$\boxed{\frac{\pi}{6} + 2k\pi \leq x \leq \frac{5}{6}\pi + 2k\pi}$

2c) $\tan x < 1$



Poichè la tangente è periodica di periodo π le soluzioni sono

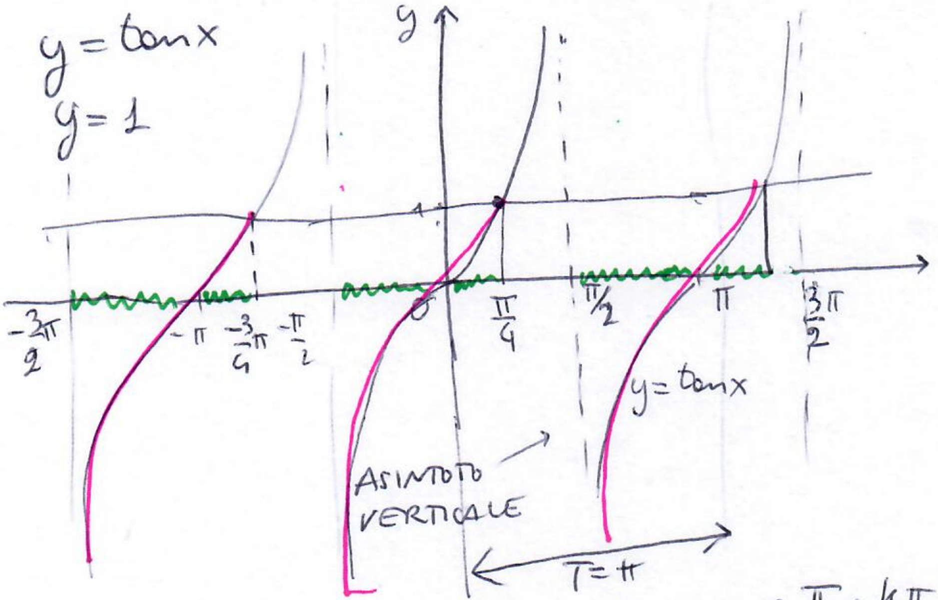
$0 + k\pi < x < \frac{\pi}{4} + k\pi$ ✓

$\frac{\pi}{2} + k\pi < x < \pi + k\pi$

GRAFICAMENTE

$y = \tan x$

$y = 1$

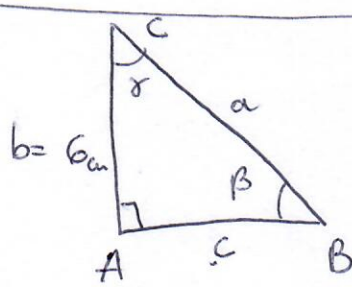


$0 + k\pi < x < \frac{\pi}{4} + k\pi \ \vee \ \frac{\pi}{2} + k\pi < x < \pi + 2k\pi$

potrei anche scrivere

$-\frac{\pi}{2} + k\pi < x < \frac{\pi}{4} + k\pi$

3



$\gamma = 20°$

$\beta = 90° - 20° = 70°$

1° TEOREMA DEI TRIANGOLI RETTANGOLI

$b = a \operatorname{sen} \beta$, $b = a \cos \gamma$

$c = a \sin \gamma$, $c = a \cos \beta$

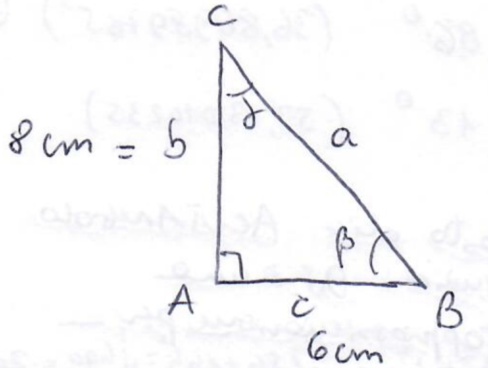
2° TEOREMA

$c = b \operatorname{tg} \gamma$ o $c = b \operatorname{cotg} \beta$

$b = c \operatorname{tg} \beta$ $\quad b = c \operatorname{cotg} \gamma$

$a = \frac{b}{\operatorname{sen} \beta} = \frac{6}{\operatorname{sen} 70°} = 6{,}385$ cm

$c = a \operatorname{sen} \gamma = 6{,}385 \cdot \operatorname{sen} 20° = 2{,}1838$ cm



TEOREMA DI PITAGORA SU $A\hat{B}C$

$$a = \sqrt{b^2+c^2} = \sqrt{8^2+6^2} = \sqrt{64+36} = 10 \text{ cm}$$

$\beta = ?$

$\gamma = ?$ $\quad a = 10$ cm

1° TEOREMA DEI TRIANGOLI RETTANGOLI

$$b = a \operatorname{sen}\beta \Rightarrow \operatorname{sen}\beta = \frac{b}{a} = \frac{8}{10} = \frac{4}{5}$$

Passo alla funzione inversa del seno per trovare l'angolo

$$\Rightarrow \beta = \operatorname{arcsen}\frac{4}{5} = 53{,}13°$$

$$\gamma = 90° - 53{,}13° = 36{,}87°$$

OPPURE → $b = a\cos\gamma \Rightarrow \cos\gamma = \frac{b}{a} = \frac{8}{10} = \frac{4}{5}$

$$\gamma = \arccos\frac{4}{5} = 36{,}869°$$

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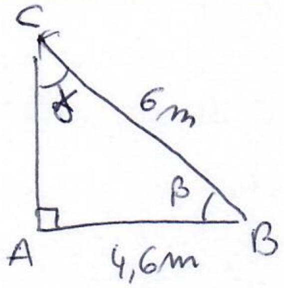
4) SCALA



$\gamma = ?$

$\beta = ?$

($\overline{CA} = \sqrt{6^2 - 4{,}6^2} = \sqrt{36 - 21{,}16} = 3{,}85$ m → questo non è RICHIESTO)

$$\overline{AB} = \overline{CB}\sin\gamma \Rightarrow \sin\gamma = \frac{\overline{AB}}{\overline{CB}} = \frac{4{,}6}{6} = 0{,}7\bar{6} \Rightarrow$$

$$\gamma = \operatorname{arcsen} 0{,}7\bar{6} \cong 50{,}06°$$

$$\beta = 90 - 50{,}06° \cong 39{,}94°$$

Approssimando a un centesimo

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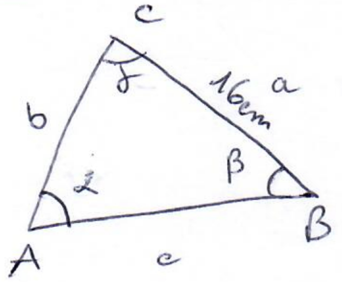
5)



$\sin A\hat{B}C = \frac{3}{5}$ → β

α $\sin B\hat{A}C = \frac{4}{5}$

$A = ?$

$2p = ?$

TEOREMA DEI SENI

$$\frac{b}{\sin\beta} = \frac{a}{\operatorname{sen}\alpha} \rightarrow b = a\frac{\sin\beta}{\sin\alpha} = \frac{16\cdot\frac{3}{5}}{\frac{4}{5}} =$$

$$b = \cancel{16}^{\,4}\cdot\frac{3}{\cancel{5}}\cdot\frac{\cancel{5}}{\cancel{4}_{\,1}} = 12 \text{ cm}$$

⑥

$\sin A\hat{B}C = \sin\beta = \frac{3}{5}$ $\quad\beta = \arcsin\frac{3}{5} = 36,87°$ $\quad(36,86989765°)$

$\sin B\hat{A}C = \sin\alpha = \frac{4}{5}$ $\quad\alpha = \arcsin\frac{4}{5} = 53,13°$ $\quad(53,13010235)$

$\gamma = 180° - 36,87° - 53,13° \cong 90°$

Il Triangolo è rettangolo e quindi: $c = \sqrt{a^2+b^2} = \sqrt{256+144} = \sqrt{400} = 20\,cm$

$2p = 12+16+20 = 48\ cm$

$A = \frac{a \cdot b}{2} = \frac{16 \cdot \cancel{12}^{6}}{\cancel{2}} = 96\ cm^2$ $\quad$ (Oppure $S = \frac{1}{2} a \cdot b \sin\gamma$

$= \frac{1}{\cancel{2}} \cdot 16 \cdot \cancel{12}^{6} \cdot \underbrace{\sin 90°}_{=1} =$

$= 96\ cm^2$)
